Modelling and Rendering Techniques for African Hairstyles

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ABSTRACT

We develop or enhance hair modelling and rendering techniques to produce three different forms of hair commonly found in African hairstyles. The forms of hair are natural curly hair, straightened hair, and braids or twists of hair.

We use an implicit model, implemented as a series of textured layers to represent curly hair. Straightened hair is represented explicitly, and modelled by defining and replicating a few control hairs. Braids and twists are implemented as textured generalized cylinders.

A synthesis of existing hair illumination models is used as a basis for an African hair illumination model. Parameter values to match African hair characteristics are discussed.

A number of complete African hairstyles are shown, demonstrating that the techniques can be used to model and render African hair successfully.

Categories and Subject Descriptors

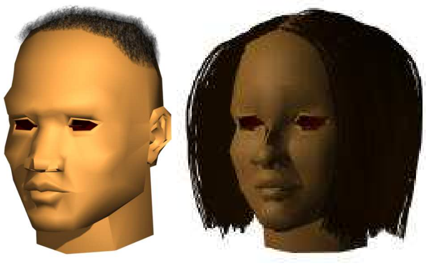
I.3.5 [**Computer Graphics**]: Computational Geometry and Object Modeling Physically based modeling; I.3.7 [**Computer Graphics**]: Three-Dimensional Graphics and Realism Color, shading, shadowing, and texture

Keywords

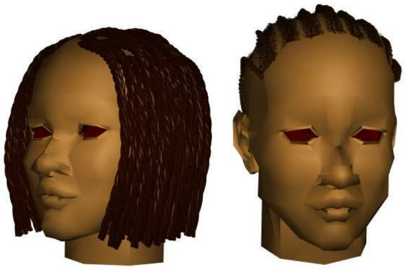
african hair, modelling, illumination

1. INTRODUCTION

Several methods have been developed for generating hair, and although some methods produce very convincing results, most methods only work well for particular types of hair under specific conditions. None of these methods specifically address the issue of generating African hair, which differs from hair of other ethnic groups. African hair is curly or kinky, contains melanin that causes it to have a dark colouring, and can be opaque due to large, dense pigment



(a) Curly (b) Straightened



(c) Twists (d) Braids

Figure 1: African Hairstyles

granules [7]. Due to the difference in hair type and cultural traditions, African hairstyles are different from hairstyles of other ethnic groups. They often contain patterns with significant numbers of braids and twists of hair [15].

In this paper we develop or enhance existing hair modelling and rendering techniques to produce different forms of hair commonly found in African hairstyles. In terms of hair modelling, we provide facilities to allow hairstyles to be created that have naturally curly, or straightened hair, or that incorporate braids and twists. In terms of hair rendering we adapt existing hair illumination models to deal with the appearance of African hair.

1.1 Modelling African Hair

Hair modelling is challenging due to the large number of hair strands, their geometric properties, and the complexity and large variation of human hairstyles. The average African scalp contains between 100,000 and 150,000 extremely thin (between 0.05mm and 0.09mm) and very curly strands of hair, which are often straightened (using a process called relaxing) or styled into braids or twists [19]. Extensions (fake hair usually in the form of braids) can also be added to the hair.

Instead of modelling individual strands of hair, it is possible to use a texture that represents strands of hair and produces the effect of hair when the image is rendered. Texturing is a useful rendering technique that solves the problem of modelling the large number of hair strands required, but does not always produce very realistic results. We make use of various texturing techniques to enhance rendering of African hair.

1.2 Illuminating African Hair

Hair rendering is challenging and computationally expensive due to the large number of hair strands, the thin geometric shape of a strand of hair, and the complex lighting effects that occur among the strands of hair. Due to the geometry and orientation of individual strands of hair, anisotropic specular highlights appear in the hair. These highlights form definite patterns across the hair, even when individual strands of hair cannot be seen. The strands of hair and the head cast shadows on each other. The colour of hair differs from person to person, and may also differ from strand to strand, or even within a strand. If the hair has a light colour (which is not the case for African hair) a haloing effect is produced under backlighting conditions due to the hair being partially transparent.

1.3 Paper Structure

Section 2 describes the state of hair modelling within the field of computer graphics and reviews the literature on existing hair modelling and hair rendering techniques. Section 3 describes the techniques used to model three identified forms of African hair. In section 4 the illumination of the hairstyles modelled with our techniques is discussed.

2. RELATED WORK

2.1 Hair Modelling Strategies

The methods available for modelling hair can be divided into three main categories: those that model hair implicitly, those that model hair explicitly, and those that use a combination of implicit and explicit techniques to model hair as clusters.

2.1.1 Implicit Models

Implicit hair models define the shape of hair using mathematical functions, providing a compact representation of the complexity of hair. It is often possible to create a variety of hair appearances with the same defining functions just by changing the values of a few parameters. Disadvantages are high computational costs, and difficulty defining appropriate functions that describe the precise appearance of a hairstyle.

Volumetric models create the appearance of hair by applying a 3D texture (or volumetric texture) of hair onto a surface. These are able to produce very convincing representations of fur or short hair, but are not appropriate for modelling long hair, where complex shapes and interactions are required. Examples of this approach include hypertexture [27] and approaches based thereon [3, 28].

Kajiya and Kay [13] extend the hypertexture approach so that 3D textures, such as fur, can be applied to objects of arbitrary shape. In each texel, a density, surface frame and bidirectional light reflectance function parameters are stored. The rectangular shape of the texels may be deformed when the texels are mapped onto a surface in order to prevent gaps or overlapping from occurring between adjacent texels.

Lengyel [21] uses a simplified volumetric model to generate short hair. His model represents hair using a set of concentric, partially transparent, textured layers displaced from the surface. The textured layers, called shells, are slices of volumetric texture. The 2D texture slices can be rendered much faster than hypertexture or texels. Papaioannou [25] extends this to include shadows cast by hairs.

Hadap and Magnenat-Thalmann [10, 11] model hair shape as streamlines of fluid flow. Due to the continuum property of fluid, hair-to-hair collision avoidance is automatically implemented and hair-to-body avoidance is described by the flow of fluid around an obstacle.

2.1.2 Explicit models

Explicit hair models are brute-force methods that model the geometry of each individual strand of hair. They allow detailed control of the hairstyle but can be complex and costly to render. The geometry of individual strands of hair can be defined using lines, curves, cylinders, or surfaces

Kong and Nakajima [17] reduce overhead by modelling fewer, thicker hairs. Thick surface hairs close to the viewer are broken down into many thinner strands of hair.

Anjyo et al. [1] model the downward bending of a strand of hair due to gravity using a simplified cantilever beam simulation with the fixed side corresponding to the root of the hair. To create cutting or combing effects they apply external forces.

Heinen and Walter [12] model each strand of hair (repre-

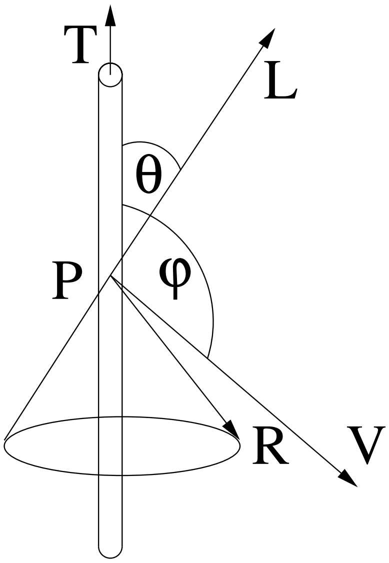


Figure 2: Geometry for Kajiya and Kay's Illumination Model.

senting fur) as a path of a particle, which is then rendered as a polyline or collection of connected cylinders. Particle systems do not explicitly model the geometry of hair strands (like most explicit hair models), however, the shape of each individual strand of hair is explicitly defined.

2.1.3 Cluster Models

Cluster (or wisp) hair models make use of the observation that strands of hair tend to form clumps in nature (especially when the hair is wet or oily). The methods explicitly model the shape of each cluster and then implicitly add details of individual strands of hair by procedurally rendering strands of hair (using volume rendering techniques), or by applying hair textures to the surfaces defining the cluster. These models are therefore useful for modelling characteristics such as braids, but are not useful for modelling curly hairstyles where the shapes of adjacent hair strands vary greatly.

Clusters simplify modelling of specific hairstyles, and are widely used for this purpose [5, 29, 32].

A widely used cluster approach [9, 16, 30] models hair in clusters using 2D strips. To create the illusion of individual strands of hair, texture maps are applied to each strip.

2.2 Hair Illumination

The most well-known illumination model used for rendering hair, which has been used by several researchers [17, 19, 30], is Kajiya and Kay's anisotropic lighting model [13]. The lighting model calculates the illumination for a thin cylinder, or segment of a single strand of hair, as shown in Figure 2.

The diffuse component, $k_d \sin(\theta)$, is obtained by integrating Lambert's Cosine Law along the circumference of the half of the cylinder visible from (and illuminated by) the light source. Self-shadowing is not considered and the hairs are fully lit even when the viewer and the light sources are on opposite sides of the hair. However, since most hair is partially translucent, the model provides an acceptable approximation of the diffuse component. The area in shadow is assumed to transmit a similar amount of light through the hair toward the viewer as that reflected from the illuminated side [19].

The specular component, $k_s \cos^n(\phi+\theta-\pi)$, is obtained using a Phong light reflection model modified for a thin cylindrical surface. Because the cylinder representing a strand of hair is so thin (usually a polyline), the surface normals at each point along the cylinder point in all directions perpendicular to the tangent vector, causing light to be reflected in all directions around the cylinder. The reflection vector is therefore a cone around the cylinder (see Figure 2).

Using the diffuse and specular components, together with an ambient component, the light intensity, Ψ_{hair}, of a point P on a hair strand can be calculated using equation (1), where L_a and L_i are the light intensities of the ambient light and each light source i respectively.

$$\begin{aligned}\Psi_{hair} &= k_a L_a + \sum \left(k_d \sin(\theta)\right) L_i + \\ &\quad \sum \left(k_s \cos^n(\phi+\theta-\pi)\right) L_i \qquad (1)\end{aligned}$$

Although Kajiya and Kay's model is a well-known illumination model that considers the anisotropic property of hair and produces reasonable results, it does not implement the effect of backlighting (i.e. haloing), which is prominent in light-coloured hair. Backlighting can be added to the model [31, 33] by keeping track of the density or thickness of the hair at each pixel.

Another weakness of Kajiya and Kay's model is that it does not consider the direction of the viewer, and hairs are fully lit even when the light source is behind the hair. Goldman [8] considers effects of both light reflection and transmission, with an attenuation factor to adjust the relative transmission and reflection of light. The amount of reflection versus transmission is similar for light-coloured hair; for dark hair the amount of reflection dominates.

Anjyo et al. [1] also introduce an anisotropic lighting model. Since the illumination model is developed for dark, glossy hair, they ignore the diffuse component. The specular component is based on the Blinn specular model [4].

Due to the anisotropic property of hair, the surface normal at each point on a hair strand points in many directions, causing an increased and more uniform intensity of light over the surface. Banks [2] modifies Kajiya and Kay's model by adding an excess brightness exponent to the diffuse component to compensate for the average increase in light intensity on objects such as hair.

Lengyel [21] presents an illumination technique that is able to illuminate hair with different viewer directions, using standard hardware-assisted lighting. Different normal vectors are used for the diffuse and specular components. Lengyel calculates a new normal as a weighted combination of these.

The latest research by Marschner et al. [24] describes a hair strand as a transparent elliptical cylinder with an absorbing interior and a surface covered with tilted scales. Their approach produces multiple specular highlights and approximates the effect caused by the rough, elliptical surface.

2.2.1 Shadowing

Strands of hair cast shadows onto the head and each other. Shadows can be generated using ray tracing or shadow maps [19]. Lokovic and Veach [23] enhance traditional shadow maps by developing deep shadow maps, which are able to generate shadows for partially transparent and volumetric objects. Kim and Neumann [14] generate shadows using a faster approximation of deep shadow maps, which they call opacity shadow maps. Instead of storing a visibility function for each pixel, opacity shadow maps approximate the decrease in light intensity for each pixel using a set of opacity maps. Other shadowing approaches include shadow masks [5] and opacity buffers [18].

Several researchers model hair with layers. Lower layers are darkened to mimic the effect of shadowing [20, 21, 25].

3. MODELLING AFRICAN HAIR

We identify three forms of hair commonly found in African hairstyles and develop techniques to model and render these different forms of hair: natural curly hair, straightened hair, and braids or twists of hair (as shown in Figure 3). We leave it up to the user to combine our techniques to create a particular hairstyle.

3.1 Natural Curly Hair

Natural African hair is short and curly, and many African people keep their hair in hairstyles consisting of the short, curly form of hair. In order to create this type of hairstyle, we need to develop a hair model that is able to produce the characteristics of short, curly hair effectively.

We choose to use an implicit hair modelling technique to model the natural, curly form of African hair. An implicit hair modelling technique seems the most appropriate technique to use for two reasons. Firstly, the hair is very curly. Curly hair requires many more polygons or segments than straighter hair, making the explicit and cluster models expensive. Secondly, a hairstyle comprised of natural curly hair tends to appear as a layer of hair over the scalp. The layer of hair has a constant thickness and pattern, analogous to a layer of fur, and fur is successfully modelled using implicit hair modelling techniques (see Section 2.1.1).

Our implicit model is based on Lengyel's concentric shell method [21], which creates the illusion of fur. Our model represents hair with concentric textured layers that can be thought of as slices of 3D texture. The layers are created parallel to the scalp one above another. An opacity map is applied to each layer, which defines what part of the layer represents hair and what part does not. Opaque areas on the textured layer represent slices of hair strands, while transparent areas represent spaces in between the hair strands.

(a) Curly

(b) Straightened

(c) Braids

Figure 3: A range of African hairstyles.

Tangent maps are also applied to the layers to control the illumination of the hair. When many layers are placed close to one another they produce the effect of hair. A further opacity map is applied to each shell to define the overall shape of the hairstyle.

The opacity maps used to define the individual strands of hair are produced by generating random curly splines in 3D space, and then taking slices of the 3D space at specific heights. The number of slices and the height at which each slice is taken depends on the number and height of the layers used. Each slice is then used as an opacity map. At the same time we generate tangent maps, which are used in our illumination model (see Section 4.1). Our method differs from that of Papaionnou [25] who creates opacity maps from greyscale images of noise (random intensity values), and is not able to produce curly hair. Lengyel [21] creates the opacity maps by growing hair with a particle system. His method is able to produce curly hair, and could have been used instead of our spline method, as could any other equivalent approach.

Hair rendered with layers, can be seen in Figure 1a. Larger numbers of layers produce better results. In this example at least 12 layers are needed to produce realistic-looking hair. Unrealistic results are obtained when the layers are placed too far apart from one another. This causes gaps to appear between the layers so that specks of hair are seen instead of the effect of strands of hair.

Use of texture maps overcomes issues with modelling a large number of hair strands. The density of the hair does not affect the rendering time because the texture maps are preprocessed. Since the textures have the same resolution, they have the same storage requirements (unless compressed). Smaller texture maps that represent hair can be tiled over a surface if they have a toroidal topology.

3.2 Straight Hair

African people often straighten their naturally curly hair using a process called relaxing. Implicit hair modelling techniques are not appropriate for modelling long hair, and cluster methods are better at modelling hairstyles where hair is grouped into clusters rather than smooth hairstyles. We therefore choose to model straight hair in African hairstyles explicitly.

The problem of a large number of hair strands is accentuated when each individual strand of hair has to be modelled. We overcome this by using control hairs to define a hairstyle, as is done by Daldegan et al. [6]. A number of control hairs are placed on the head model. These control hairs are used to describe a hairstyle, and to define the shape and geometric properties of hair strands in their area on the head model. To create a full head of hair more curves are automatically defined and placed on the head model using the shape and geometric properties of the control hairs. By using a few control hairs that define a hairstyle, many strands of hair can be modelled automatically, as shown in Figure 4.

This hairstyle is defined by 68 control hairs and takes approximately $2\frac{1}{2}$ hours to design. The final hairstyle is produced by automatically creating 40 copies of each control

(a) Control hairs

(b) Complete hairstyle

Figure 4: Explicitly modelled hairstyles with straight hair.

hair.

Sometimes a few hair strands are created that have an unwanted shape or appear in the wrong place. Fortunately, this is easily solved because the hair strands are explicitly modelled. By selecting and moving control points of the stray hairs, the shape of the hairs can be modified, and unwanted hairs can be selected and deleted.

Although a large number of polygons are needed to represent hair explicitly, there is an upper bound on the number of polygons needed to produce a reasonable hairstyle. Generating more hair strands than needed will increase the time taken to render the hair without improving the visual quality of the hair. The number of hairs needed is well below the number actually present on a human head [26].

3.3 Braids and Twists of Hair

Many common African hairstyles contain hair that has been styled into the form of braids or twists. We use a cluster hair modelling technique to model hair in the form of braids and twists. Only the shape of the wisps of hair that form braids and twists are modelled, rather than each individual strand of hair. This reduces the complexity and time required to model braids and twists of hair.

Our cluster model uses generalized cylinders to model the wisps of hair that make up a braid or twist of hair. Texture maps are applied to the generalized cylinders to create the effect of a cluster of individual strands of hair.

The control curves for twists and braids are defined by extruding circles (2), and Lissajous figures (3) respectively.

$$\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin(\theta) \\ z &= k\theta \end{aligned} \tag{2}$$

$$\begin{aligned} x &= r\cos(\theta) \\ y &= \frac{r}{f}\sin(2\theta) \\ z &= k\theta \end{aligned} \tag{3}$$

The tightness of the twist is controlled by parameter k, while the parameter f allows the braid to be flattened. Generalized cylinders are then extruded along these control curves. The use of braids and twists is shown in Figure 5.

Texture maps can be applied to the generalized cylinders that represent clusters of hair to create the effect of many strands of hair. The texture maps may be images of hair strands, or they may be generated procedurally.

Sometimes flatly parted hair on the scalp can be seen in African hairstyles that contain braids and twists. This hair is not part of the braids and twists, but forms the part of the base and is layered against the scalp. To create the effect of parted hair that can be seen between braids and twists in a hairstyle, we place a cap on the scalp [22]. However, instead of placing darkly coloured caps onto the scalp, we place texture maps representing tessellated patterns of hair, as shown in Figure 5c.

4. ILLUMINATING AFRICAN HAIR

The results obtained using a simple illumination model on generalized cylinders would be appropriate for African hair. However this is impractical due to the large number, complex shape, and narrow width of strands of hair. Instead we use a hair illumination model based on Kajiya and Kay's anisotropic lighting model [13], applied to simpler geometry. However, a number of researchers have made improvements to different components in Kajiya and Kay's model (see Section 2.2), and we combine several of these improvements to develop our model.

The ambient component, $k_a L_a$, is calculated as usual.

The diffuse component is calculated using Kajiya and Kay's diffuse term with the excess brightness exponent presented by Banks [2] as implemented by Lengyel [20, 21]:

$$k_d \left(1 - (T \cdot L)^2\right)^{\frac{k}{2}}$$

Here T and L represent the tangent and light vectors respectively (see Figure 2), and k is the excess brightness exponent. Banks recommends a value of 4.7635 for k.

African hair usually has a very dark colour and is often black. If the colour of the hair is set to black, the diffuse component has no effect and is not needed in the illumination model.

The specular component, as implemented previously by Anjyo et al. [1] is:

$$k_s \left(1 - (T \cdot H)^2\right)^{\frac{n}{2}}$$

Here n is the Phong exponent, H is the halfway vector between L and V (the eye vector).

For light-coloured hair (e.g. blonde hair) the expressions above provide an acceptable approximation of the light intensity at a point, even though the direction of the viewer is not considered, because the hair is partially transparent and transmits light through the hair. However, African hair contains dense pigment granules that cause it to be far more opaque than other types of hair [7]. African hair therefore does not transmit light through the hair, and the direction of the viewer with respect to the direction of the light needs to be taken into account in the equation above.

We use a directional attenuation factor [8], f_{dir}, (see (4)) to modulate the diffuse and specular components.

$$f_{dir} = \left(\frac{1+\kappa}{2}\right)\rho_{reflect} + \left(\frac{1-\kappa}{2}\right)\rho_{transmit} \tag{4}$$

Here $\rho_{transmit}$ and $\rho_{reflect}$ vary in the range $[0, 1]$ and represent the amounts of forward and backward scattering, and κ represents the relative directionality of L, V and T, specifically the cosine of the angle between the vectors $T \times L$ and $T \times V$ (where $\times$represents the cross product). When L and V are on the same side of the hair there is front lighting, and $\kappa > 0$.

(a)

(b)

(c)

Figure 5: Hairstyles with braids and twists.

When rendering black, opaque African hair, our illumination model does not need a diffuse component and the directional attenuation factor could be implemented as a constant value (with $\rho_{reflect} = 1$ and $\rho_{transmit} = 0$). However, African hair does not always have a totally black surface colour, and a user may want a certain amount of light to be transmitted through the hair. Our illumination model with a diffuse component and directional attenuation factor allows African hair, with different colours and opacities, to be illuminated, and can also be used to illuminate other types of hair.

Other features that could be added to our illumination model include backlighting and a variation in colour between strands of hair or within a strand of hair. Since African hair is opaque it does not transmit light and haloing is not as prominent under backlighting conditions [19]. Also the colour of adjacent strands of hair, or within an individual strand of hair, tends to be the same. A vertex colour map could be used to vary the colour within a strand of hair.

Adding shadows to hair is important in order to create realistic looking hair, as shown in Figure 6. We use traditional shadow maps to add shadows to our explicitly modelled hair, merely because they have faster rendering speeds than other shadowing techniques, and because the facility is available in the rendering engine. These provide adequate results, although if the resolution of the shadow map is too low, the quality of the image produced is poor. For the images shown in this paper, the difference in quality for different shadow map resolutions is negligible after a resolution of 2048 x 2048 pixels.

4.1 Illuminating Textured Layers

The illumination model above can also be used to add lighting effects to hair that is modelled using textured layers (discussed in Section 3.1). Since the hair strands are represented by a texture rather than explicitly, the tangent vector for each hair strand is stored in a texture map. The shader needs to reference this tangent map, rather than use a value calculated from the layer geometry.

To add the effect of self-shadowing in the hair we assume the hair represented by layers closer to the scalp is in more shadow than hair further away from the scalp. We make the surface colour of the lower layers closer to the scalp darker than the colour of higher layers further away from the scalp. Since only the surface colour of the layers is changed, adding self-shadowing to hair represented by textured layers has no effect on the rendering time.

Figure 7 shows the effects of adding shadowing and lighting to short curly hair.

5. CONCLUSION

Our implicit, explicit and cluster hair modelling techniques can be used to create a variety of different African hairstyles with relative ease. Different types of hair can be created for the same hairstyle by applying different texture maps, and different hair densities can be created with texture maps without affecting the rendering time, because the texture maps are pre-processed. A large number of polygons are needed to represent hair explicitly or with clusters. However,

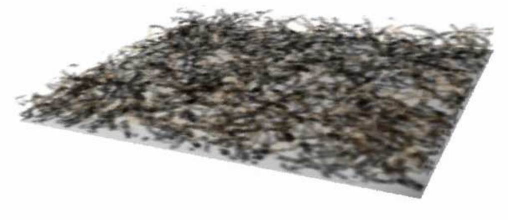
(a) Standard Lambert/Phong illumination applied to cylinders

(b) Hair illumination model applied to line primitives

(c) With shadow mapping

Figure 6: Illumination of straight hair.

(a) No illumination



(b) With self-shadowing

(c) With illumination

Figure 7: Illumination of texture layers.

for most of the variables there is an upper bound on the number of polygons needed to produce a reasonable hairstyle.

We describe a hair illumination model and discuss the effects that the nature of black, opaque African hair has on the model. In particular, effects of diffuse reflection and transmission through hair can be ignored. We leave them in the illumination model so that hair with different colours and opacities can be rendered.

Different African hairstyles created with our hair modelling and rendering techniques are shown in Figure 1. The hairstyles presented demonstrate that our techniques can be used to simulate African hairstyles that contain curly hair, straight hair, and braids or twists of hair.

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